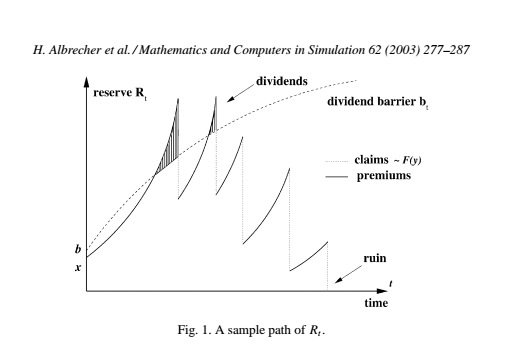
**Introduction**

For an insurance company, a ruin occurs when a claim size is greater than the reserves. The goal for insurance company before was thus to minimize the probability of ruin. However, it is clear that a senior company that starts business earlier has a smaller probability of ruin when facing a same claim than a new company only because more premium has been collected. In other words, there is no need to hold as many reserves as possible. Therefore, the modern insurance companies pay out the excessive part to shareholders as dividends when the reserves are greater than certain amount, which is called barriers.

The companies follow different strategies when paying out dividends. For instance, in barrier strategy, dividends are paid continuously at a rate when surplus reaches some levels and stop when new claims happen and the reserves drop below the barrier (Figure 1). This strategy shows what an insurance do when they have earned enough profit from customers.

In the band strategy, the dividends are paid according to the region where X(t) is located. It is similar to threshold strategy, but it is the discrete case whereas the threshold strategy is continuous case. And the barrier strategy is a special case of the band strategy.



Articles that we consulted include *On the Time Value of Ruin* by Hans U. Gerber A.S.A.,

Ph.D. & Elias S.W. Shiu A.S.A., Ph.D.,*The classical risk model with a constant dividend barrier: analysis of the Gerber–Shiu discounted penalty function* by X. Sheldon Lin a,∗, Gordon

E. Willmot b, Steve Drekic b, and *Simulation methods in ruin models with non-linear dividend barriers* by Hansjörg Albrecher∗, Reinhold Kainhofer, Robert F. Tichy. Through which, we learned Gerber and Shiu Function, optimal barrier calculation, band strategy and non-constant barrier simulation methods.

This research intends to discover the ruin probability and expected present value (EPV) of total dividend with different constant, linear, and exponential barriers, and test using Monte-Carlo simulation.

**Mathematical Approach for Ruin Probability and EPV Calculation**

We used the approach from the article *On the Time Value of Ruin* to analyze the ruin probability and expected present value of dividends. We calculated the ruin probability without barrier and EPV of dividends with constant barrier. First, we assumed that the insurer’s initial surplus is u which is larger or equal than 0 and the premiums are received continuously at a constant rate, c, per unit time. The aggregate claims constitute a compound Poisson process, {S(t)}, given by the Poisson parameter lambda and individual claim amount is under exponential distribution P(x) = mu\*exp(-mu\*x) with P(0) = 0, x>0. By integrating ruin probability by parts and differentiating them, we simplified the ruin probability. Then we introduced some new parameters and Laplace Transform which helped us get a much simpler approach to the ruin probability. After analyzing our distribution by these methods, we finally got our own ruin probability which is lambda\*exp(u\*(lambda/c-mu))/(c\*mu).

For the EPV of dividends, by consulting the same article under optimal dividend strategies, we got an idea that no dividends are paid unless the surplus reaches the level b before ruin occurs. Also by conditioning on the time and amount and differentiating them, we got our EPV formula which has exponential distribution for individual claim amount. More detailed calculations for ruin probability and EPV of dividends are in the Appendix.

For insurance companies, if we know the actual numbers of the parameters in the formulas, we can analyze the real mathematical results of their ruin probability and EPV of dividends. Also, these mathematical approach formulas help us compare with the numerical results from MATLAB.

**Ruin Probability Simulation with Barriers**

In comparison to the constant barrier strategy which has been analyzed by others, we considered a barrier adjusted with respect to time. Then what we need is to add another parameter which varies with different time. Since we pay out dividend to the shareholders as time goes, the interest rate always matters. We then find out that interest rate is an ideal parameter in our barrier model.

We choose a compound Poisson risk model with mean λ for the time of claim occurrence.

We assume the claim size is exponential with mean μ. We set up an initial reserve u, an initial barrier b, a constant premium rate c and the force of interest i. Considering inflation, we choose our barrier as b(t)=b\*exp(i\*t). A ruin occurs when u(t)=u+c\*t is less than the claim size x(t) if

u(t) is less than b(t), or b(t) is less than x(t) when u(t) is greater than b(t). We also assume that at a certain time t, all claims are treated as one claim. In this model, we will find the ruin probability with t< 1000, and the corresponding expected present value of dividends.

Function Pruin is created to calculate the ruin probability. Variables created in this

function include initial barrier b, initial reserve u, barrier’s force of interest i, compound Poisson risk model mean λ, claim size mean μ and premium rate c. We first simulate the time interval of claim occurrence using dt = −log(rand(1,1))/lambda because it follows exponential distribution. We initialize t to be 0, so the time of occurrence can be expressed as t = t + dt. We then simulate the process using a while loop when t is less than 1000. In every occurrence of claim, we first simulate a claim size by generating a random exponential number with mean mu using function exprnd(). Then we compare u (t) with b (t) to decide if barrier has been reached. If it does, the reserve is equal to b (t), and is equal to u (t) it doesn’t. Then we compare the reserves with the claim size. If the claim is greater than the reserves at time t, the while loop terminates, and the indicator variable p is assigned with value 1, indicating a ruin occurred; otherwise, if no ruin happens during the while-loop, 0 is assigned to p, indicating no ruin occurs. Function Loopsum runs the procedure above for 100 times and sum all p values. Dividing the sum of p values by 1000, we then derive the probability of ruin. There is a flow chart demonstrating the process in appendix.

*Fig 2: Flow chart illustration for ruin probability simulation*

**Monte-Carlo Simulation for EPV of Dividend**

1. **Exponential barrier**

We simulate the process with the chosen barrier in MATLAB using Monte-Carlo method. Function EPV is used to calculate the expected present value of dividend. Similar to the method for calculating ruin probability, in every claim occurrence, we first calculate u(t) and

b(t). Then we solve equation. If u(t) is greater than u(t), there should be one solution t0 before the time of occurrence (Figure 3). So the EPV is calculated as , where delta is the discount rate.

Barrier b

EPV

Reserve u

t0 x t0+dt time

*Fig 3: the EPV integration is from solution x to t0+dt*

If u(t) is less than b(t), the only case that a dividend exists is when there are two solutions x1, x2 for equation The EPV of dividends is thus calculated as (Figure 4). If a ruin occurs during the loop (u < 0), the loop terminates and no more EPV will be added.

Barrier b

Reserve u

t0 x1 x2 t0+dt time

*Fig 4: There are two solutions between t0 and to+dt, the shaded area are paid dividend*

For expected present value of dividends, we choose lambda, mu, c, u, b, i as 0.5, 100, 10, 2000, 5000, and 0.0006. Lower lambda and b and i and larger c and u make EPV become larger. From total surplus graph, we can see that EPV is largest when the barrier is constant which means i = 0.

From the results we got from Matlab, when i = 0, the expected present value of dividend is largest. That is, our barrier strategy is not an optimal dividend strategy since EPV of dividend is less than that of constant dividend strategy. Also, our barrier increases as t increases. This means that the reserve should be larger to reach the larger barrier level as time goes in order to get dividend to pay. If reserve is not large enough to reach the barrier level, then there is no dividend paying to shareholders. Hence, our barrier strategy is not an optimal dividend strategy.

1. **Comparison with Other Model Assumptions**
2. Ruin Probability Comparison

The ruin probability and EPV of dividends depend on the barrier strategy. When barrier is different, we expect that ruin probability and EPV of dividends will change a lot. We would like to consider following barrier strategy: no barrier, constant barrier, linear barrier and exponential barrier. First, we consider our time is exponential distributed and we let lambda, mu, c, u, b, delta be 0.4, 1/80, 35, 500, 1000, 0.05.

With the help from computer simulation, we obtained data under the circumstances without dividend barrier and with a constant barrier. We took lambda, mu as variables and kept other constant. The ruin probabilities are as shown below:

1. EPV of Dividends Comparison

Due to the computation capacity, we set out time to be 1000 year but the theoretical time is infinity. Therefore, our theoretical result is greater than the simulation as expected. We also simulated several dividend-paid model, and the constant barrier model with various lambda, Mu.

Note that the fourth graph has a constant barrier 1000.

1. Different Barrier Strategy Comparison

We compared EPV with three different barrier strategies, which are constant, exponential and linear to see how different barrier strategies can influence EPV. We set lambda=0.4, mu=80, c=40, u=500, b=1000, delta =0.05, the result are show in the flowing table:

|  |  |
| --- | --- |
|  | EPV |
| Constant Barrier | 94.1543 |
| Exponential Barrier (i=0.03) | 0 |
| Linear Barrier (k=5) | 37.1208 |

Analyzing the result, we found that for linear barrier and exponential barrier, the increase rate is critical to determine the EPV of dividends. If the increase rate is set too large, there is little change for the reserve u exceeds the barrier, which should result in a small EPV of dividends. Especially for exponential barrier, since the barrier increases at a much higher speed, once a large claim happens, hardly can the reserve to “catch up” with the barrier again. The simulated EPV is not quit consistent with the theoretical value, and there is a high variation when doing the simulation. We believe this is due to the number of times simulated.

1. Constant Barrier with Different Gamma Time Intervals Comparison

We change the time interval to Gamma distribution. We intend to compare how change of parameters in the time interval model can result in change in EPV of dividends. Therefore, we set mu, c, u, b, delta constant, and only change the Gamma parameter α and β. Since we set the mean of exponential distribution time interval to 1/0.4=2.5, we fix the mean of Gamma distribution at 2.5. The results are shown in the table below. When mean is constant, the EPV of dividends increases when variance of Gamma distribution increases. By analyzing the value of time interval dt, we find when variance is small, dt varies in a small scale around 2.5, which result in an almost constant increase in reserve u between every claim. If a severe claim occurs, it will either lead to a ruin or a significant decrease in reserve u. In such case, since the increase between every claim is almost constant, there is less change for the accumulated reserve u to exceed the barrier. In order words, high variance in time internal has higher possibility to lead to high accumulated reserve u, which can potentially cancel out the possibility of a severe claim.

|  |  |  |  |
| --- | --- | --- | --- |
| Models | Mean(αβ) | Variance(α\*β^2) | EPV of Dividends |
| Constant Barrier (α=0.625, β=8) | 2.5 | 20 | 994.5095 |
| Constant Barrier (α=1.25, β=2) | 2.5 | 5 | 758.5993 |
| Constant Barrier (α=2.5, β=1) | 2.5 | 2.5 | 65.9824 |
| Constant Barrier (α=5, β=0.5) | 2.5 | 1.25 | 56.2394 |
| Constant Barrier (α=10, β=0.25) | 2.5 | 0.625 | 42.9773 |